

Graphing Rational Functions

A **rational function** are functions where $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are both polynomials and $b(x) \neq 0$.

$f(x)$ has a **vertical asymptote** whenever $b(x) = 0$

$f(x)$ has at most one **horizontal asymptote**

- If the degree of the numerator is greater than the denominator, there is no horizontal asymptote.
- If the degree of the numerator is less than the denominator, the horizontal asymptote is the x-axis ($y = 0$)
- If the degree of the numerator equals the denominator, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$

Points of discontinuity (hole) – Occurs when the numerator and denominator have a common factor.

Examples:

1. Describe the asymptotes and points of discontinuity of $f(x) = \frac{x^2+4x-5}{x+5}$

2. Given the function: $g(x) = \frac{(x-2)(3x+2)}{(x+4)(x-2)(x-6)}$

- What are the equations of the asymptotes of this function?
- Determine if there are any points of discontinuity. Explain why or why not.
- Describe the end behavior as x approaches $-\infty$ and as x approaches $+\infty$